two-dimensional stream function ψ are

$$[\psi_y(\partial/\partial x) - \psi_x(\partial/\partial y) - (1/R)\nabla^2]\nabla^2\psi = 0 \quad (1)$$

An inner expansion, valid within the jet, is assumed as

$$\psi(x,y;R) = R^{-1/2}\Psi_1(x,Y) + R^{-1}\Psi_2(x,Y) + \ldots \quad (2)$$

where $Y = R^{1/2}y$. The equation for Ψ_1 is

$$\Psi_{1YYY} + \Psi_{1x}\Psi_{1YY} - \Psi_{1Y}\Psi_{1xY} = 0 \quad (3)$$

The boundary conditions are

$$\Psi_1(x,0) = \Psi_{1Y}(x,0) = 0 \text{ and } \Psi_{1Y}(x,\infty) = 0$$

A similarity solution for Ψ_1 has been found by Glauert.[1] We write

$$\Psi_1(x,Y) = 4x^{1/4}f(\eta) \text{ where } \eta = Y/x^{3/4}$$

The problem for f is $f''' + ff'' + 2(f')^2 = 0, f(0) = f'(0) = 0$, and $f'(\infty) = 0$. The solution is

$$\eta = \log_e\left[\frac{(1 + f^{1/2} + f)^{1/2}}{1 - f^{1/2}}\right] + (3)^{1/2}\tan^{-1}\left[\frac{(3f)^{1/2}}{2 + f^{1/2}}\right] \quad (4)$$

Flow Due to Displacement

The first-order inner viscous solution induces a correction in the outer inviscid solution. An outer expansion, valid outside the jet, is assumed as

$$\psi(x,y;R) = 0 + \delta_2(R)\psi_2(x,y) + \ldots \quad (5)$$

Matching with the first-order inner solution yields $\delta_2(R) = R^{-1/2}$ and provides the boundary condition for the displacement problem. It is $\psi_2(x,0) = \Psi_1(x,\infty) = 4x^{1/4}$, $x > 0$. ψ_2 satisfies Laplace's equation and the additional boundary condition that $\psi_2(x,0) = 0$ for $x < 0$. The solution is

$$\psi_2(x,y) = 4\{\mathrm{Re}(x + iy)^{1/4} - \mathrm{Im}(x + iy)^{1/4}\} \quad (6)$$

where Re and Im denote the real and imaginary parts, respectively, for $0 \leq \arg(x + iy) < 2\pi$.

Second-Order Jet Solution

The equation for Ψ_2 is

$$\Psi_{2YYY} + \Psi_{1x}\Psi_{2YY} - \Psi_{1Y}\Psi_{2xY} + \Psi_{2x}\Psi_{1YY} - \Psi_{2Y}\Psi_{1xY} = 0 \quad (7)$$

The boundary conditions at the wall are $\Psi_2(x,0) = \Psi_{2Y}(x,0) = 0$. Matching with the second-order outer solution yields the third boundary condition $\Psi_{2Y}(x,\infty) = -x^{-3/4}$.

The similarity solution $\Psi_2(x,Y) = g(\eta)$ is assumed. The resulting problem is

$$g''' + fg'' + 5f'g' = 0$$
$$g(0) = g'(0) = 0, \; g'(\infty) = -1 \quad (8)$$

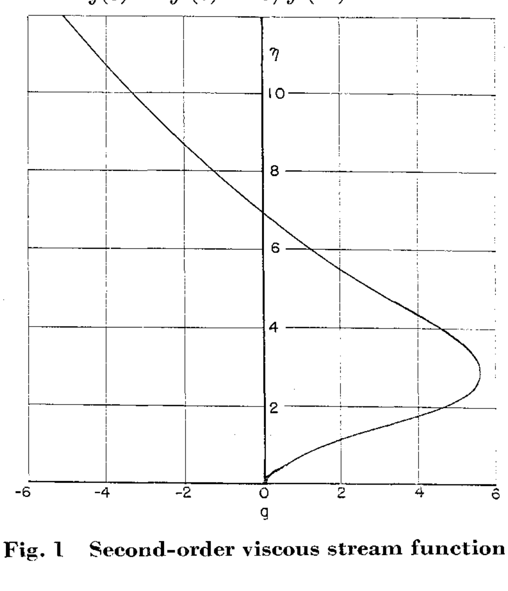

Fig. 1 Second-order viscous stream function.

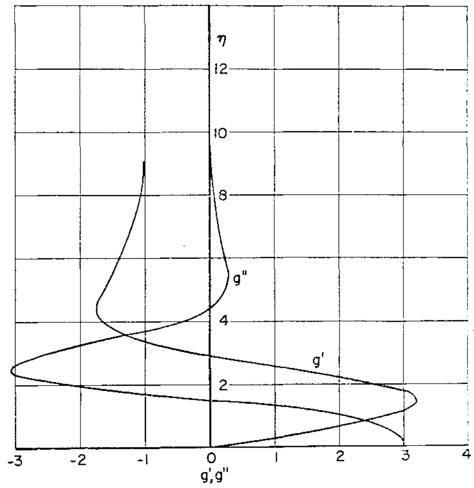

Fig. 2 Derivatives of second-order viscous stream function.

This equation has been solved numerically using a variable-step Adams-Moulton procedure. The results are shown in Figs. 1 and 2.

The nondimensional shear stress at the wall is given by

$$C_f = \tau/\tfrac{1}{2}\rho U^2 = 2[4x^{-5/4}f''(0)R^{-1/2} + x^{-3/2}g''(0)R^{-1}] \quad (9)$$

where ρ is the density. $f''(0)$ is given by Glauert as $\frac{2}{9}$ and $g''(0)$ is determined as 3 in the present calculations. The effect of displacement is to increase the shear stress at the wall.

A major obstacle to the numerical solution of the second-order problem of a jet flowing around an arbitrarily curved surface is that to begin the solution, initial profiles must be specified. If the surface of interest is initially flat, the solution presented here would appear to yield the best initial profiles now available.

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Apparent Reverse Transition in an Expansion Fan

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TURBULENT separated flows have received considerable attention in the separated flows research program at Rutgers. Part of this program includes the theoretical and

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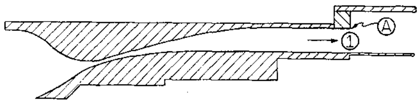

Fig. 1 Emil Buehler Wind Tunnel with expansion test section attached.

experimental investigations of supersonic separation from a two-dimensional corner. This research continues to suggest a dampening of the general turbulence level during the expansion process from the approaching boundary layer to the initial portion of the free shear layer. Recent experimental evidence has been obtained which very dramatically and clearly illustrates the apparent partial relaminarization of a turbulent boundary layer undergoing separation from a corner.

The flow of interest to this Note is generated (Fig. 1) at corner A in a special test section attached to the nozzle of a variable Mach number blowdown type supersonic wind tunnel. This facility known as the Emil Buehler Wind Tunnel has a two-dimensional nozzle exit with a width of 4.0 in. and a height at cross section (1) of approximately 4.5 in.

Figure 2 is a reproduction of a schlieren photograph recently taken with the single pulse of a Q-switched ruby laser giving an exposure of 15 nanosec. M_1 was 3.05 and the Reynolds number at (1) was 2.1×10^7 per ft. The approaching boundary layer had a n power-law parameter of 8.53 with a momentum thickness of 0.040 in. It completely fills the lower left image of Fig. 2.‡ In this short duration photograph, the turbulence level of the guided boundary layer as it approaches the corner is clearly shown. Also clearly shown is the apparent reverse transition which takes place as the expansion fan interacts with the turbulent boundary layer.

Figures 3 and 4 are reproductions of schlieren-interferometer photographs taken recently with the single pulse of a ruby laser giving an exposure of 15 nanosec. In such photographs, the fringe shift quantitatively represents the density gradient normal to the undisturbed fringe direction.[1] Again, the quenching of the guided boundary-layer turbulence by the expansion fan is clearly shown.

Theoretical calculations[2] have indicated the success of rotational characteristics in predicting downstream flowfields from initial conditions provided by a large approaching turbulent boundary layer. The experimental evidence of Figs. 2, 3, and 4 support the conjecture that under certain separating conditions the Reynolds' stresses of the turbulent boundary layer may be reduced or eliminated.

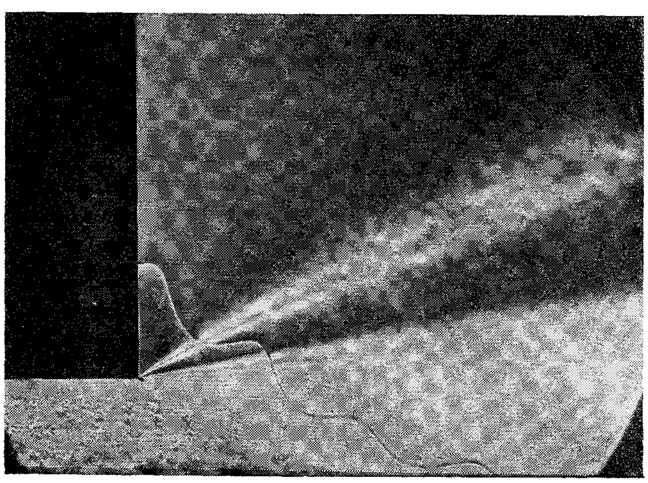

Fig. 2 Schlieren photograph of corner flow (15 nanosec exposure—horizontal knife edge).

‡ The wavy line traveling from the corner wall to the bottom of the photograph in a sine wave-like fashion is due to an imperfection in one of the glass side walls of the tunnel.

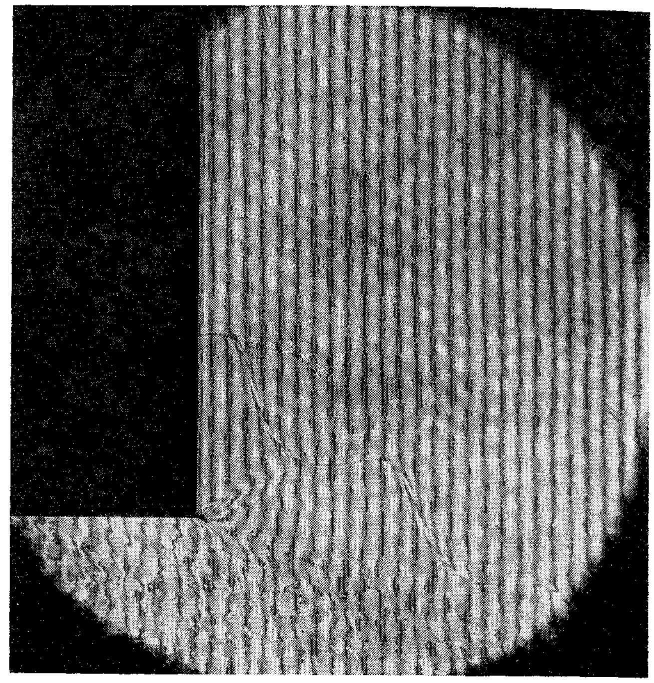

Fig. 3 Schlieren-interferometer photograph of corner flow (15 nanosec exposure—vertical fringes).

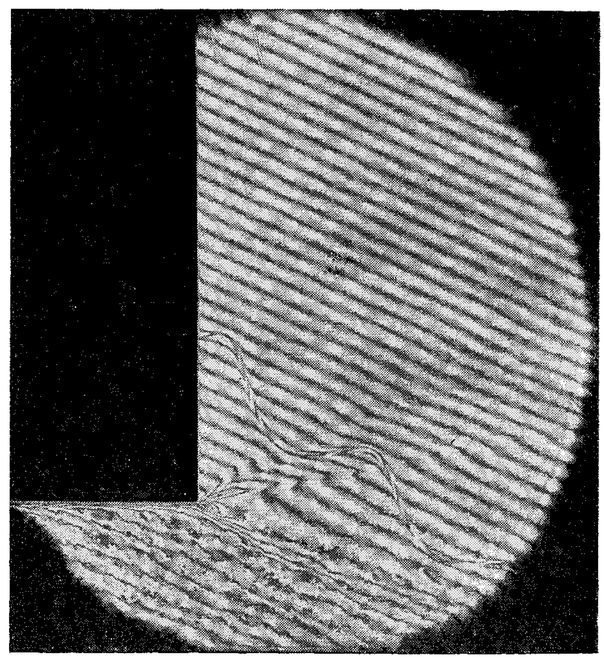

Fig. 4 Schlieren-interferometer photograph of corner flow (15 nanosec exposure—fringes 25° from horizontal).

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